Calculus 1
Midterm Exam
September 30, 2026 (18:30 – 20:30)

university of groningen

Please read the instructions!

1) Prove using the (ε, δ)-definition that $\lim_{x \to a} \sqrt{x} = \sqrt{a}$ if $a > 0$.

2) Apply l'Hospital's rule to evaluate the following limit:

$$\lim_{x \to \infty} \left[x - x^2 \ln\left(\frac{1+x}{x} \right) \right].$$

Indicate which rules of differentiation are being used in each step.

3) Use differentiation to find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$.

4) Suppose that $f'(x) \geq 3$ for all $x \in \mathbb{R}$. Show that $f(2) + 18 \leq f(8)$.

5) Evaluate the indefinite integral $\int \sqrt{1 - x^4}\, dx$ as an infinite series.

6) Prove the following reduction formula for all $n \in \mathbb{N}$:

$$\int (\ln x)^n \, dx = x(\ln x)^n - n \int (\ln x)^{n-1} \, dx.$$

(Hint: Use integration by parts.)

Instructions

- **write your name and student number on the top of each sheet of writing paper!**
- use the writing (lined) and scratch (blank) paper provided, raise your hand if you need more paper
- start each question on a new page
- use a pen with black or blue ink
- do not use any kind of correcting fluid or tape
- any rough work should be crossed through neatly so it can be seen
- this is a closed-book exam (books & notes are not permitted)
- you may use your own handwritten formula sheet (a 2-sided A4 paper) – the invigilators will check these
- you are also allowed to use a simple pocket calculator
- programmable/graphing calculators are not allowed
- your work should be clearly and logically structured
- **explain your reasoning using words**
- show all your calculations, an answer without any computation will not be rewarded
- each problem is worth 15 points
- upon completion* remain seated and raise your hand to submit your work

*At the end of the exam or after you finished whichever is sooner.